Flow formulas

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1 Correlation functions

q-vectors are defined as follows:

$$u_i^n = \begin{pmatrix} \cos n\phi_i \\ \sin n\phi_i \end{pmatrix} \tag{1}$$

$$q_a^n = \frac{\sum\limits_{i=1}^{N_{tr}^a} w_i u_i^n}{\sum\limits_{i=1}^{N_{tr}^a} w_i} \tag{2}$$

$$\langle q_a^n q_b^n \rangle = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \left\{ \frac{\sum\limits_{k=1}^{N_{tr}^a\ i} w_k u_k^n}{\sum\limits_{k=1}^{N_{tr}^a\ i} w_k} \right\} \left\{ \frac{\sum\limits_{k=1}^{N_{tr}^b\ i} w_k u_k^n}{\sum\limits_{k=1}^{N_{tr}^a\ i} w_i} \right\} \tag{3}$$

Integral form (for simplicity we assume weights to be equal to 1):

$$u^n(\phi) = \begin{pmatrix} \cos n\phi \\ \sin n\phi \end{pmatrix} \tag{4}$$

$$\frac{dN_a}{d\phi} = \frac{dN_a}{d(\phi - \Psi_m)} = \frac{N_a}{2\pi} \left(1 + 2 \sum_k v_k\{\Psi_m\} \cos(k(\phi - \Psi_m)) \right) \tag{5}$$

$$\begin{aligned} q_a^n &= \int_0^{2\pi} d\phi_a u^n(\phi_a) \frac{dN_a}{N_a d\phi_a} = \frac{1}{2\pi} \int_0^{2\pi} d\phi_a \left(u^n + 2u^n \sum_k v_k \cos(k(\phi_a - \Psi_m)) \right) = \\ &= \frac{1}{\pi} \int_0^{2\pi} \begin{pmatrix} \cos n\phi_a \sum_k v_k \cos(k(\phi_a - \Psi_m)) \\ \sin n\phi_a \sum_k v_k \cos(k(\phi_a - \Psi_m)) \end{pmatrix} = \\ &= \frac{1}{\pi} \int_0^{2\pi} \begin{pmatrix} v_n \{\Psi_m\} \cos^2(n\phi_a) \cos(n\Psi_m) \\ v_n \{\Psi_m\} \sin^2(n\phi_a) \sin(n\Psi_m) \end{pmatrix} = v_n\{\Psi_m\} u^n(\Psi_m) \end{aligned} \tag{6}$$

$$\langle q_a^n q_b^n \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\Psi_m q_a^n\{\Psi_m\} q_b^n\{\Psi_m\} = \langle v_a^n v_b^n \rangle \tag{7}$$

2 Additional material

2.1 Jacobian determinant

Here we oparate in momentum space (3 dimentions for now). To perform transitions of the coordinate systems one must use Jacobian matrix. We need exactly:

$$\int d^3p = J \int dp_t d\eta d\varphi, \tag{8}$$

Where J - is the determinant of Jacobian matrix J_{ik}. For transformation $\int d^i u = J \int d^k v$ it would be:

$$J = \det\left(J_{ik}\right),\ J_{ik} = \begin{pmatrix} \frac{\partial u_1}{\partial v_1} & \frac{\partial u_1}{\partial v_2} & \dots & \frac{\partial u_1}{\partial v_k} \\ \frac{\partial u_2}{\partial v_1} & \frac{\partial u_2}{\partial v_2} & \dots & \frac{\partial u_2}{\partial v_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial u_i}{\partial v_1} & \frac{\partial u_i}{\partial v_2} & \dots & \frac{\partial u_i}{\partial v_k}. \end{pmatrix}. \tag{9}$$

In our case we have transformation (8):

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \xrightarrow{(8)} \begin{pmatrix} p_t \\ \eta \\ \varphi \end{pmatrix}, \qquad \begin{cases} p_x = & p_t \cos(\varphi) \\ p_y = & p_t \sin(\varphi) \\ p_z = & p_t \frac{e^{\eta}-e^{-\eta}}{2} \end{cases}, \tag{10}$$

p_t - transverse momentum, η - pseudorapidity, φ - azimuthal angle.

Now derive Jacobian coefficients:

$$J_{33} = \begin{pmatrix} \frac{\partial p_x}{\partial p_t} & \frac{\partial p_x}{\partial \eta} & \frac{\partial p_x}{\partial \varphi} \\ \frac{\partial p_y}{\partial p_t} & \frac{\partial p_y}{\partial \eta} & \frac{\partial p_y}{\partial \varphi} \\ \frac{\partial p_z}{\partial p_t} & \frac{\partial p_z}{\partial \eta} & \frac{\partial p_z}{\partial \varphi} \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & 0 & -p_t \sin(\varphi) \\ \sin(\varphi) & 0 & p_t \cos(\varphi) \\ \frac{e^{\eta}-e^{-\eta}}{2} & p_t \frac{e^{\eta}+e^{-\eta}}{2} & 0 \end{pmatrix}, \tag{11}$$

and obtain determinant J:

$$\begin{aligned} J = \det\left(J_{33}\right) = \begin{vmatrix} \cos(\varphi) & 0 & -p_t \sin(\varphi) \\ \sin(\varphi) & 0 & p_t \cos(\varphi) \\ \frac{e^{\eta}-e^{-\eta}}{2} & p_t \frac{e^{\eta}+e^{-\eta}}{2} & 0 \end{vmatrix} = \cos(\varphi) \begin{vmatrix} 0 & p_t \cos(\varphi) \\ p_t \frac{e^{\eta}+e^{-\eta}}{2} & 0 \end{vmatrix} - \\ -0 \begin{vmatrix} \sin(\varphi) & p_t \cos(\varphi) \\ \frac{e^{\eta}-e^{-\eta}}{2} & 0 \end{vmatrix} + \left(-p_t \sin(\varphi)\right) \begin{vmatrix} \sin(\varphi) & 0 \\ \frac{e^{\eta}-e^{-\eta}}{2} & p_t \frac{e^{\eta}+e^{-\eta}}{2} \end{vmatrix} = \\ = \cos(\varphi)\left(-p_t \frac{e^{\eta}+e^{-\eta}}{2} p_t \cos(\varphi)\right) - p_t \sin(\varphi)\left(-\sin(\varphi) p_t \frac{e^{\eta}+e^{-\eta}}{2}\right) = \\ = -p_t^2 \frac{e^{\eta}+e^{-\eta}}{2}. \end{aligned} \tag{12}$$

Thus, transformation (8) is given by:

$$\int d^3p = -\int p_t^2 \frac{e^{\eta}+e^{-\eta}}{2} dp_t d\eta d\varphi. \tag{13}$$